Interactive Traffic Simulation

Microscopic Open-Source Simulation Software in Javascript

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Traffic and congestion phenomena belong to our everyday experience. Our traffic simulation software `www.traffic-simulation.de` allows for controlling traffic and creating traffic jams in order to understand the principles behind traffic breakdowns and how to resolve them. The fast visualization illustrates how traffic jams can be seen as collective phenomena with waves propagating backwards from the bottleneck, and standing waves pinned at the beginning of the bottleneck.

The simulator offers an intuitive and interactive way of playing with the factors influencing the collective dynamics of traffic like initial and boundary conditions, inhomogeneities of the road network (onramps, offramps, lane closures, uphill grades), traffic regulations (speed limits, traffic lights, etc.), and the parameters describing the driving and lane-changing behavior of single vehicles. In the following, we describe the used car-following model for the acceleration behavior, the lane-changing model and the numerical procedure to solve the system of coupled non-linear differential equations efficiently. Finally, the default simulation scenarios are characterized and explained shortly.

1 Modeling Traffic – The Math behind the Simulation

The software uses the *Intelligent Driver Model* (IDM) to simulate the longitudinal dynamics, i.e., accelerations and braking decelerations of the drivers.[1] In such models, also called *car-following models*, the decision of any driver to accelerate or to brake depends only on his or her own speed and on the position and speed of the 'leading vehicle' immediately ahead. In contrast, lane-changing decisions depend on all neighboring vehicles. The IDM and a lane-changing model derived from it (cf. Sec. 1.2) completely describe all immediate actions a driver can do at any time, namely accelerating, braking, and steering as a response to own preferences and the local traffic environment.

More generally, the car-following and lane-changing models used in the simulator belong to the class of *microscopic traffic flow models*, in contrast to *macroscopic traffic flow models* describing traffic flow as a fluid in terms of local density and local speed. This has some analogies in the physics of fluids (liquids and gases): Microscopically, fluids are described by the interactions of their constituting molecules, and macroscopically, by the Navier-Stokes equations. Moreover, in both fields, the microscopic dynamics produce macroscopic *emergent phenomena* that do not depend on the microscopic details. The classic examples are the sound waves of fluids and the corresponding traffic waves that can be investigated by playing with the simulator. In contrast to the fluid molecules,

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[1] In fact, the IDM has been slightly modified for a 'cooler' behavior after cut-ins of other vehicles but this does not change the general idea.

the 'vehicle-driver units' of the microscopic traffic flow models are *active* or *self-driven particles* [1, 2] also called *agents*, so these models also also examples of *multi-agent models*.

1.1 The Intelligent Driver Model

The Intelligent Driver Model (IDM) is probably the simplest complete and accident-free model producing realistic acceleration profiles and a plausible behavior in essentially all (single-lane) traffic situations.[2] Its structure can be described as follows:

- The influencing factors (model input) are the own speed v, the bumper-to-bumper gap s to the leading vehicle, and the leader's speed v_l or, equivalently, the approaching rate (relative speed) $\Delta v = v - v_l$.
- The model output is the acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$ chosen by the driver for this situation.
- The model parameters describe the driving style, i.e., whether the simulated driver drives slowly or fast, careful or reckless, anticipatively or short-sighted, and so on.

The IDM acceleration equation reads as follows:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a_{\text{free}} + a_{\text{int}} = a\left[1 - \left(\frac{v}{v_0}\right)^{\delta}\right] - a\left(\frac{s^*(v,\Delta v)}{s}\right)^2. \tag{1}$$

The acceleration $\frac{\mathrm{d}v}{\mathrm{d}t}$ consists of two parts. The first part for free flow depends only on the ratio v/v_0 between the current and the desired speed producing a smooth acceleration profile with a maximum given by the model parameter a at zero speed, and a zero acceleration at the desired speed. The second part (which is always negative) describes the repulsive 'social force' exerted by the leading vehicle. Similarly to the repulsive force of two equally charged particles, this force is proportional to $1/s^2$ – with the difference that the interaction is unilateral: The leader is not 'pushed' by the follower. This term compares the desired gap s^* to the current gap s. The desired gap

$$s^*(v,\Delta v) = s_0 + \max\left(0, vT + \frac{v\Delta v}{2\sqrt{ab}}\right) \tag{2}$$

has a steady-state ('equilibrium') term $s_0 + vT$ and a *dynamical* term $v\Delta v/(2\sqrt{ab})$ that implements the 'intelligent' braking strategy [2]. Notice that, if the actual gap is approximatively equal to s^*, the breaking deceleration essentially compensates for the free acceleration part, so the resulting acceleration is nearly zero. This means, $s_0 + vT$ corresponds to the gap when following other vehicles in steadily flowing traffic. In addition, s^* increases dynamically when approaching slower vehicles and decreases when the front vehicle is faster. As a consequence, the imposed deceleration increases with

- decreasing distance to the front vehicle (one wants to maintain a certain *safety distance*),
- increasing own speed (the safety distance increases),
- increasing approaching rate to the front vehicle (when approaching the front vehicle at a too high rate, a dangerous situation may occur).

The IDM has intuitive parameters with the values used for the simulation summarized in Table 1. In general, every 'driver-vehicle unit' can have its individual parameter set. For example, *trucks* are characterized by low values of v_0, a, and b, careful drivers drive at a high safety time headway T, and aggressive ('pushy') drivers are characterized by a low T in connection with high values of v_0, a, and b. Often two different types are sufficient to show the main phenomena.

[2] For a more detailed description of the IDM we refer to the Chapter 11 of our book [2]. This is freely available from `www.traffic-flow-dynamics.org/res/SampleChapter11.pdf`.

Parameter	Car	Truck	Remark
Desired speed v_0 when driving on a free road	120 km/h	80 km/h	For city traffic, one would adapt the desired speed while the other parameters essentially can be left unchanged
Desired safety time headway T when following other vehicles	1.5 s	1.7 s	Recommendation in German driving schools: 1.8 s; realistic values vary between 2 s and 0.8 s, and even below
Minimum bumper-to-bumper gap s_0 to the front vehicle	2 m	2 m	Kept at complete standstill, also in queues that are caused by red traffic lights
Acceleration a in everyday traffic	$0.3\,\mathrm{m/s^2}$	$0.3\,\mathrm{m/s^2}$	Very low values to enhance the formation of stop-and go traffic. Realistic values are 1-2 $\mathrm{m/s^2}$
Comfortable (braking) deceleration b in everyday traffic	$3.0\,\mathrm{m/s^2}$	$2.0\,\mathrm{m/s^2}$	Very high values to enhance the formation of stop-and go traffic. Realistic values are 1-2 $\mathrm{m/s^2}$
Acceleration exponent δ	4	4	

Table 1: Model parameters of the Intelligent Driver Model (IDM) used in the simulation.

1.2 The Lane-Changing Model MOBIL

Lane changes take place if another lane is more attractive ('incentive criterion'), and the change can be performed safely ('safety criterion'). In our lane-changing model MOBIL [3] we base both criteria on the accelerations in the old and the prospective new lanes, as calculated with the longitudinal model (that is the IDM in the simulation).

The *safety criterion* is satisfied if the IDM braking deceleration $-a^{\mathrm{IDM}}$ imposed on the new follower f' of the target lane after a possible change does not exceed a certain limit b_{safe}, this means, the safety criterion fulfills

$$a_{\mathrm{f}'}^{\prime,\mathrm{IDM}} > -b_{\mathrm{safe}}. \tag{3}$$

In this formula, the acceleration $a_{\mathrm{f}'}^{\prime,\mathrm{IDM}}$ stands for the IDM acceleration of the new follower f' caused by the lane-changing vehicle *after a prospective change* (the dash in a' denotes the acceleration after a change).

In order to asses the *incentive criterion*, we ask whether we, as a driver, obtain some advantage by a lane change *in terms of an increased acceleration.* Furthermore, we do not take the risk of lane changing for a marginal benefit and model this by imposing a *changing threshold* Δa_{thr}. Finally, we also include a bias Δa_{bias} to the left or right lanes caused, e.g., by the desire or need to merge or diverge soon, or simply by the 'keep-right directive' of many European countries. In quantitative terms, the incentive criterion is satisfied if

$$a'^{\;\mathrm{IDM}} > a^{\mathrm{IDM}} + \Delta a_{\mathrm{thr}} \pm \Delta a_{\mathrm{bias}}, \tag{4}$$

where a^{IDM} and $a'^{\;\mathrm{IDM}}$ denote the IDM acceleration of the subject driver before and after the change, respectively.

1.3 Mathematical Structure and Numerical Solution

The mathematical form of the IDM model equations (1) and (2) is that of *coupled ordinary (non-linear) differential equations*:

- They are differential equations since, in one equation, the dynamic quantities $v(t)$ (speed) and its derivative $\frac{dv}{dt}$ (acceleration) appear simultaneously.

Parameter	Typical Value	Remark
Max. safe deceleration b_{safe}	$4\,\text{m/s}^2$	Must be lower than maximum deceleration of about $9\,\text{m/s}^2$
Threshold a_{thr}	$0.2\,\text{m/s}^2$	Must be below the lowest acceleration ability (IDM parameter a) of any vehicle type
Bias to the right lane Δa_{bias}	$0.2\,\text{m/s}^2$	The absolute value must be higher than Δa_{thr}

Table 2: Model parameters of the lane-changing model MOBIL used in the simulation. For mandatory lane changes, $|\Delta a_{\text{bias}}| = 5\,\text{m/s}^2$

- They are coupled since, besides the speed v, the equations also contain the speed $v_{\text{lead}} = v - \Delta v$ of the leading vehicle. Furthermore, the gap s obeys its own kinematic equation, $\frac{\mathrm{d}s}{\mathrm{d}t} = -\Delta v$ coupling, again, the (time derivative of the) gap to the leading speed.
- They are non-linear due to the square term and power of δ in Eq. (1) and the product as well as the maximum condition in Eq. (2). Under suitable conditions (ring road with an obstacle thrown on the road), the model's nonlinearity allows even to simulate *deterministic chaos*.

Simulation means to numerically 'integrate', or, in other words, approximatively solve the coupled differential equations of the model. Specifically, we consider a finite and fixed numerical update time interval Δt, and integrate over this interval assuming constant accelerations. This so-called *ballistic method* reads[3]

$$\text{new speed:} \quad v(t+\Delta t) = v(t) + \frac{\mathrm{d}v}{\mathrm{d}t}\Delta t, \tag{5}$$

$$\text{new position:} \quad x(t+\Delta t) = x(t) + v(t)\Delta t + \frac{1}{2}\frac{\mathrm{d}v}{\mathrm{d}t}\Delta t^2. \tag{6}$$

where $\frac{\mathrm{d}v}{\mathrm{d}t}$ is the IDM acceleration calculated at time t, and x is the position of the front bumper. For the IDM, any update time step below $0.5\,\text{s}$ (or, $\lesssim T/2$, respectively) will essentially lead to the same result, i.e., sufficiently approximates the true solution. Strictly speaking, the model is only well defined if there is a leading vehicle and no other object impeding the driving. However, generalizations are straightforward:

- If there is no leading vehicle and no other obstructing object ('free road'), just set the gap to a very large value such as $1000\,\text{m}$. The limit of the gap tending to infinity is well-defined for any meaningful car-following model such as the IDM.
- If the next obstructing object is not a leading vehicle but a red *traffic light* or a stop-signalized intersection, just model these objects by a standing virtual vehicle of length zero positioned at the stopping line. In order to simulate a transition to a green light, just eliminate the virtual vehicle.
- If a speed limit becomes effective, reduce the desired speed, if the present value is above this limit. Likewise, reduce the desired speed of trucks in the presence of gradients.

Special Case of Stopped Vehicles. For vehicles approaching an already stopped vehicle or a red traffic light, the ballistic update method as described above will lead to negative speeds whenever the end of a time integration interval is not exactly equal to the true

[3]For a discussion of different numerical schemes to solve these equations and why the ballistic method is the most suited one we refer to the paper [4].

stopping time (of course, there is always a numerical mismatch). Then, the ballistic method has to be generalized to simulate following approximate dynamics:

> If the true stopping time is within an update time interval Δt, decelerate at constant deceleration $\frac{dv}{dt}$ to a complete stop and remain at standstill until this interval has ended.

Furthermore, it may happen that the actual gap of a stopped vehicle s is slightly below the minimum gap s_0, in which case the IDM would give a negative acceleration, hence a negative speed in the next time step. In most cases, however, real drivers will just keep that somewhat too low gap until the leader drives again rather than driving backwards. Both special cases can be implemented by following rules:[4]

$$\text{if} \quad v(t) + \frac{dv}{dt}\Delta t < 0, \text{ then} \tag{7}$$

$$\text{new speed: } v(t+\Delta t) = 0, \tag{8}$$

$$\text{new position: } x(t+\Delta t) = x(t) - \frac{1}{2}v^2(t)/\frac{dv}{dt}. \tag{9}$$

1.4 Boundary conditions

A coupled system of ordinary differential equations does not only need initial conditions but also *boundary conditions*. In our simulator, we use two categories:

1.4.1 Periodic boundary conditions

When simulating ring roads and other closed systems, there are no boundary conditions in the strict sense. However, formally, one has to 'cut' the ring at one position which is then the set of virtual upstream and downstream boundaries. For a ring of circumference L, this means that $x_i \to x_i - L$ whenever vehicle i exceeds the position $x = L$, and the gap for the first vehicle is given by $s_{\text{first}} = x_{\text{last}} - l_{\text{last}} - x_{\text{first}} + L$ which depends on the 'last' vehicle, i.e., that with the smallest value of x.

1.4.2 Open boundary conditions

In contrast to closed roads with periodic boundary conditions that are controlled by the density, open systems are controlled by the flow, specifically,

- the upstream boundary determines the flow and the system in free conditions,
- the downstream boundary determines the flow in congested conditions.

The implementation of open boundary conditions can be tricky:

- *Upstream boundary conditions*: Integrate the inflow Q_{in} over the simulation time, $n_{\text{buffer}} = \int_0^t Q_{\text{in}}(t')dt'$, and, as soon as the vehicle number n_{buffer} in the upstream buffer exceeds 1, try introducing a vehicle in the simulation and decrement n_{buffer} by 1. In congested conditions, this is not always successful reflecting the fact that then the downstream boundary counts.
- *Downstream boundary conditions*: These are not so simple since just taking away vehicles according to the integrated downstream flow condition brings in artifacts if the vehicles to be removed have not yet reached the boundary. It is better to set the *speed* of the most downstream vehicles (which no longer have a leader) to the prescribed boundary speed. If the speed is low enough, this allows introducing congestions via the downstream boundary. For free-flow conditions, the fixed speed has no influence on the dynamics and corresponds to 'free boundary conditions' (vehicles leave the simulation without leader as though the road is free).

[4]Notice that $-\frac{1}{2}v^2/\frac{dv}{dt}$ is greater than zero, if this special case applies.

2 Simulation Scenarios

In the following, we describe the simulation scenarios, the main user interactions and observable traffic phenomena.

2.1 Simulation Scenario: Ringroad

This simulation scenario, depicted in Fig. 1), shows multi-lane vehicular traffic in a *closed system* (ring road). We simulate two types of vehicles, *cars* and *trucks* which are distinguished by the IDM parameters given in Table 1.

The dynamics depends essentially on the red sliders, namely on the *average vehicle density* ρ which is the main control parameter in closed systems, and on the IDM parameter a characterizing the agility (responsiveness) of the drivers.

- In the standard settings, traffic flow is unstable and backwards moving traffic waves appear after some time. This is caused by the dense traffic and simultaneously sluggish driver settings: a follower responds too late to small braking maneuvers of the leader (caused, e.g., by a lane change) and consequently closes in too much. In order to re-obtain the desired gap $s_0 + vT$, the follower has to decelerate *even more*. The same applies to the next follower, and so on. Eventually, this 'vicious cycle' results into a fully developed traffic wave with a region of stopped vehicles.
- You can click on a vehicle imposing a controlled perturbation to study this mechanism.
- For higher densities, several waves appear.
- For densities below 25 vehicles per km and lane, the leader is able to accelerate before the next follower closes in and the 'vicious cycle' is broken: No traffic waves appear
- The same is true when increasing the IDM acceleration a thereby making the drivers more responsive: Even developed traffic waves resolve after some time!
- You can also reduce the number of lanes to 1 ('freeway minus' symbol) and/or eliminate the trucks (truck percentage to zero) to realize that neither lane changes nor driver-vehicle heterogeneity are relevant factors for this mechanism.[5]
- By dropping a stationary construction vehicle onto the road, you can transform the travelling traffic waves into standing waves.
- Drop a traffic light onto the road and switch it to green after some time. Observe that the queue of waiting vehicles does not dissolve instantaneously but from the front at the same velocity as that of traffic waves
- You can also elongate the road by pulling it into hairpins, figure-of-eight shapes, and combinations thereof.

As bottomline, we learn from this ring scenario the following:

- Traffic waves always propagate *against* the direction of the traffic flow at a velocity of about 15 km/h which does not depend on the system size, the initial or boundary conditions, the perturbations, or the traffic context (city, country road, freeway, change the free-flow speed for that). This is a sort of *universal traffic flow constant*. It is observed in real-world traffic worldwide (cf. right plot in Fig. 1 from Ref. [5]).
- The outflow of all types of moving downstream fronts of congested traffic (per lane) is about the same. This includes stop-and-go waves but also dissolving queues in city traffic when the traffic light gets green. This so-called *dynamic capacity* is by typically 10-20 % lower than the static capacity of the road. The resulting *capacity*

[5] Cf. the provided documentary video available from `https://www.youtube.com/watch?v=azmcu1cn2vg`

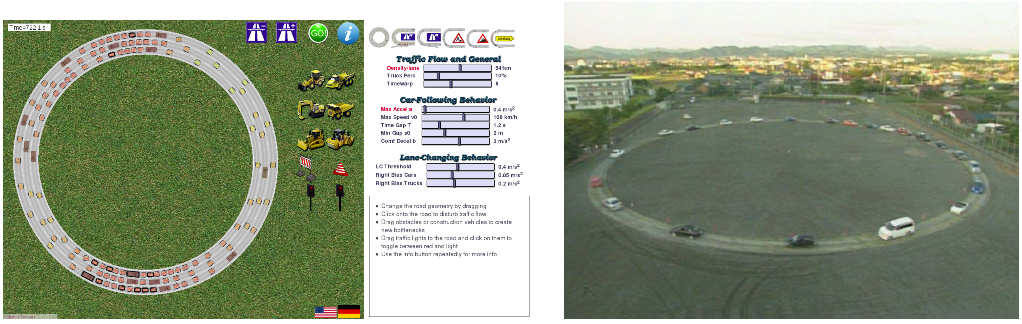

Figure 1: Left: Simulation screenshot for an increased density; right: a real-world experiment [5].

drop is the reason why avoiding a traffic flow breakdown is crucial and traffic jams resolve so slowly.

2.2 Open System Scenarios with Stationary Bottlenecks

The simulator provides four scenarios with open boundaries and several forms of infrastructure bottlenecks (Fig. 2): On-ramp, off-ramp, roadworks (lane closing), and an uphill section.

- With the initial settings of the respective simulation, traffic breaks down at or near the bottleneck region which, then, triggers upstream propagating traffic waves.
- Once the waves have formed, you can slow down the simulation speed and click at an entering vehicle to observe how it encounters seemingly ‘phantom’ traffic waves.
- As in the ring scenario, reduced traffic (controlled by the inflow rather than the density) and a higher driver’s responsiveness will make the waves (but not necessarily the congestion) disappear.
- Also *off-ramps* may act as bottlenecks even though traffic *leaves* the road, so, naively, one could think of an ‘anti-bottleneck’.
- Change the speed limit in the lane closing scenario and observe that traffic does not break down at the initial setting (limit 80 km/h) but for higher (and also lower!) speed limits. Notice the strong capacity drop in this case.
- Even locally changed driving characteristics, e.g., at curves or uphill sections, may serve as a bottleneck. Reduce the maximum speed a truck can drive at the uphill section and play with the truck overtaking ban.
- Before a breakdown has occurred, click on a vehicle in the bottleneck region to apply a disturbance. Notice how this vehicle triggers a breakdown although the driver may not even notice it!

As bottomline, we learn from these scenarios the following:

- In reality, phantom jams are not really ‘phantom’ but they have a cause and that cause is the bottleneck downstream: Because of its invisibility (the bottleneck may be several kilometers downstream) and the apparent causality (the driver encounters the effect before the cause) the illusion of a phantom jam appears: There is always a weakest ink.
- The bottlenecks come in many forms. Their common and defining aspect is a local decrease of the road capacity, i.e., its maximum throughput without causing congestions.
- Generally, we have *three ingredients to make a jam*: High traffic demand, a bottleneck, and a local disturbance, e.g., caused by a lane changing or by the user clicking on a vehicle.

Figure 2: Screenshots of the open-system scenarios with different bottlenecks. The onramp represents a non-flow-conserving bottleneck. The flow-conserving bottlenecks differ in their strength: The lane closure is a strong inhomogeneity while the uphill grade is a mild one affecting only the trucks.

- Traffic managing measures such as speed limits try reduce the local disturbances and thereby prevent/delay a breakdown. One could say that *slower is faster* or, regarding ramp metering, *less is more*.

2.3 Routing Game

In the final 'deviation' scenario of the simulator, you can simulate the effects of routing recommendations issued, e.g., by navigation devices or variable message signs on the road. Play with the *deviation use* slider and notice that one can do too much of a good thing: Instead of a mainroad jam behind the lane closing, traffic on the deviation may break down. Since the deviation route has a much lower capacity, a congestion on it will take much more time to resolve.

Play the ‘routing game’ and try to control traffic flow by the *deviation use* slider with the objective of bringing all vehicles (there is only a fixed number) through the simulation in the shortest time!

Figure 3: Screenshot of the simulation scenario ‘Routing Game’.

Lessons learnt. Navigation and routing may be tricky since there is a delay between the decision point (taking the main road or the deviation) and the consequences (traffic jam on either road). Generally, a feedback control with delays tends to be unstable. In our case, if you do not watch out carefully, you will cause *routing oscillations*.

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